Hydrodynamic Model Calibration for a Mesotidal Lagoon: the Case of Ria de Aveiro (Portugal)

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ABSTRACT


Ria de Aveiro is a shallow vertically homogeneous mesotidal lagoon, located in the Northwest of Portugal. It has a very intricate morphology and large tidal flats areas that cover and uncover along the tidal cycle. A two-dimensional vertically integrated hydrodynamic model was considered to be adequate to simulate its hydrodynamics and a model developed from the SIMSYS2D model was applied. The purpose of this study is to describe the implementation and calibration of this model which is able to predict tide-induced water level and depth integrated velocity for the entire lagoon. The numerical bathymetry used was developed from data concerning depth obtained from a general survey carried out in 1987/88 and actualised for several channels in the last years. The Monte Carlo method was used to generate the rectangular computational grid, which has 409 cells in the eastward direction and 966 cells in the northward direction (dimensions of 40×40 m). The model calibration was performed adjusting the bottom friction coefficient for the entire lagoon, through the comparison between measured and predicted time series of sea surface elevation (SSE) for a large number of stations distributed along the main channels of the lagoon. The relative mean absolute error, the root-mean square and a skill assessment of the difference between those SSE values were determined to quantify the models performance. Harmonic analysis was also performed in order to evaluate the model accuracy. According to the results the hydrodynamic model was successfully calibrated, reproducing accurately the barotropic flows in this complex lagoon.

ADDITIONAL INDEX WORDS: Numerical modelling, Harmonic analysis, Skill assessment

INTRODUCTION

The mathematical modelling of physical processes in shallow waters is a very complex process that has been developed during the last decades and actually still continues in great development. The two-dimensional vertically integrated (2DH) models have already achieved a full development state being able, for example, to be used in the study of estuaries and lagoons. A mathematical model is by definition an attempt to approximate and reproduce real phenomena (CHENG et al., 1991). The approximations and parameterisations used for the synthesis of the model lead to discrepancies and deviations of model results from nature. Therefore, before being applied to a specific location, the models should be calibrated. Model calibration appears in various forms, dependent on data availability, characteristics of water body and most of all the perceptions and opinions of modellers (HSU, 1999).

In this study it is applied a two-dimensional vertically integrated hydrodynamic model developed from the SIMSYS2D model (LEENDERTSE and GRITTON, 1971; LEENDERTSE, 1987). This model was previously applied to Ria de Aveiro (DIAS and LOPES, 2006a,b), using a numerical grid with cells of 100×100 m. The application presented in this work consists in the use of a refined grid, with cells 40×40 m, in order to better reproduce the narrow channels of this system.

The main aims of this paper are to perform the hydrodynamic model calibration for Ria de Aveiro lagoon and to contribute to establish a methodology to generally perform this task.

STUDY AREA

Ria de Aveiro (Figure 1) is a shallow vertically homogeneous lagoon with a very complex geometry, located on the northwest coast of Portugal (40º38'N, 8º45'W). It is 45 km long and 10 km wide and covers an area of 83 km² at high water (spring tide) which is reduced to 66 km² at low water (DIAS and LOPES, 2006b). It is characterised by narrow channels and by large areas of mud flats and salt marshes.

Ria de Aveiro is a mesotidal lagoon (DAVIES, 1964) and the tides, which are semidiurnal, are the main forcing action. Tidal amplitude at the inlet ranges from a minimum value of 0.6 m in neap tide to a maximum of 3.2 m in spring tide with an average value of 2 m (DIAS et al., 2000). The lagoon receives freshwater from two main rivers, Antuã (5 m³/s average flow) and Vouga (50 m³/s³) (MOREIRA et al., 1993).

The estimated maximum and minimum tidal prism of the lagoon is 136.7×10⁶ m³ and 34.9×10⁶ m³, respectively, for the maximum range spring tide and the minimum range neap tide. The total estimated freshwater input in a spring tide is very small (about 1.8×10⁶ m³ during a tidal cycle) when compared with the mean tidal prism at the mouth (about 70×10⁶ m³) (DIAS and LOPES, 2006). These values suggest a homogeneous vertical structure, as referred in a former hydrological lagoon classification (DIAS et al., 1999).
HYDRODYNAMIC MODEL

A two-dimensional vertically integrated hydrodynamic model based on finite differences techniques was applied. This model was developed from the SIMSYS2D model (Leendertse and Gritton, 1971; Leendertse, 1987) and solves the second order partial differential equations for the depth-average fluid flow derived from the full three-dimensional Navier-Stokes equations. This results in a system consisting one equation for the mass vertically averaged by integrating from the bottom to the surface:

\[ \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \left[ \rho \frac{h + \zeta}{C} \right] + \frac{\partial}{\partial y} \left[ \rho \frac{h + \zeta}{C} V \right] = 0 \]  

(1)

and

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = fV - g \frac{\partial \zeta}{\partial x} - \frac{t^+_x}{(h + \zeta) \rho} + A_n \nabla^2 U \]  

(2)

where \( U \) and \( V \) are the depth integrated velocity components in the \( x \) (eastward) and \( y \) (northward) directions, respectively, \( \zeta \) is the surface water elevation, \( h \) is the water depth, \( t \) is the time, \( f \) is the Coriolis parameter, \( g \) is the acceleration of gravity, \( \rho \) is the water density, \( A_n \) is the kinematic constant turbulent horizontal viscosity, \( t \) is the magnitude of the shear stress on the bottom.

The bottom stress is assumed proportional to the square of the horizontal velocity (Dronkers, 1964; Leendertse and Gritton, 1971):

\[ t^+_x = \rho_0 g \frac{U(U^2 + V^2)^{1/2}}{C^2} \quad \text{and} \quad t^+_y = \rho_0 g \frac{V(U^2 + V^2)^{1/2}}{C^2} \]  

(4)

where \( C \) is the Chézy coefficient. This coefficient depends on the bottom roughness and composition and on the height of the water column. In the present work the Chézy coefficient is determined from the Manning roughness coefficient, \( n \) (Chow, 1959):

\[ C = \frac{C_h}{n} \]  

(5)

The system of Eqs. (1)–(3) was discretised using a finite difference method and is solved by implicit finite difference techniques, with the variables defined on a space-staggered rectangular grid (Leendertse and Gritton, 1971). A 'fractioned-step' technique combined with an Alternating Direction Implicit (ADI) algorithm is used in the solution of the difference equations to avoid the necessity for iteration.

In order to ensure second order accuracy the variables were defined as follows: the water levels \( \zeta \) are computed at integer values of the grid; the Chézy values are computed at the same locations, whereas the depths values \( h \) are given at half-integer values of the grid; the velocity components are situated between the locations of the water levels (the \( u \) velocity components are computed at half-integer values of \( x \) and integer values of \( y \), and the velocities \( v \) are computed at half-integer values of \( y \) and integer values of \( x \)).

The ADI algorithm implies that at each time step a solution is first made in the \( x \)-momentum equations followed by a similar solution in the \( y \)-direction. The application of the implicit finite difference scheme results in a tridiagonal system of equations for each grid line in the model. The solution is obtained by inverting the tridiagonal matrix using the Double Sweep algorithms, a very fast and accurate form of Gauss elimination. The implicit scheme is used in such a way that stability problems do not occur provided, of course, that the input data is physically reasonable so that the time step used in the computations is limited only by accuracy requirements.

With appropriate boundary and initial conditions, this system of equations constitutes a well-posed initial boundary value problem whose solution describes the depth-averaged circulation in a tidal basin. For barotropic models driven by tidal forcing the boundary conditions from experimental data include both the reflected and the incident waves, therefore and by its simplicity are adopted extrapolation formulas at open boundaries in this model. A dynamic water elevation at the ocean open boundary was imposed. It was defined as the water elevation over the reference level and was determined from the harmonics constituents obtained for the tide gauge located at station D (Figure 1) and corrected in amplitude and phase to the open boundary location. The initial conditions were horizontal level and null velocity in all the grid points. Along the solid boundaries a null normal velocity was imposed and a free slip condition was assumed. In this study it was not taken into account the wind and freshwater induced flows,
since they were found irrelevant in the previous modelling efforts in Ria de Aveiro (DIAS and LOPES, 2006a,b).

The numerical bathymetry used (Figure 1) was developed from data concerning depth obtained from a general survey carried out in 1987/88 by the Hydrographic Institute of the Portuguese Navy and actualised for several channels in the last years. The Monte Carlo method was used to generate the rectangular computational grid, which has the dimensions $\Delta x = 40\ m$ and $\Delta y = 40\ m$, resulting in 409 cells in the $x$ direction and 966 cells in the $y$ direction. It was adopted values of 20 $s$ and 20 $m^2/s$ for the time step and for $A_k$, respectively.

Flooding and drying occur frequently in several zones of the computational domain: salt-marshes and tidal flats. This was simulated by making the location of the land-water boundary a function of the current value of the depth, thus simulating the changing boundary during rising and falling tides. It is therefore essential that the models yield stable and correct solutions when floods are physically and leads to numerical problems that arise as a result of the discretised representation of this hydrodynamic process which generally varies in a smooth manner.

In this model the methodology adopted is presented by LEENDERTSE and GRITTON (1971) to control flooding and drying conditions, which was already successfully used in several different environments. These authors define a volume element of water associated with each grid point at each time level and suggested the determination of flow transport between adjacent grid points.

The Chézy coefficient, $C$, is used as an indicator to designate whether a particular grid point is dry ($C=0$) or wet ($C>0$). There are several procedures by which a grid point that is currently wet can become dry. The first check is made after each calculation of a new water level. If the new computed value of the water level has decreased so that a negative volume is obtained, then the correspondent grid point is considered dry and is taken out of the computational domain. The Chézy coefficient and the current values of the velocity components leading to that grid point are set to zero and the previous row or column is recalculated with the point now taken as dry in order to correctly simulate the new situation in the computation of adjacent water levels and velocities. As a point is allowed to become dry the water elevation for the grid square are equated to the corresponding values at the end of the previous half time step when the cell was last wet. Thus conservation of water mass throughout the computation field is maintained. All flow processes are eliminated in these areas.

The second check on the possibility of a point becoming dry is performed after the calculation of the water levels and velocities. Transport cross sections are first computed using the newly calculated water levels from the flow computation. A check is performed on all four transport cross sections associated with each of the four adjacent grid points. A negative cross section is impossible physically and leads to transport in the direction opposite to what it should be. Therefore, if any of the cross sections are negative the point is considered dry and the values at these points and all velocities leading to these points are set to zero. Again a thin layer of water equal to the current value for the water level at these points is allowed to remain to assure mass conservation.

The above two checks are made at each time step to avoid incorrect transport of water, therefore discrete changes in the system boundaries can occur at each time step. The solutions obtained with the finite-difference model are solutions with variables changing gradually in time and space. If discrete changes are made, local discontinuities are generated which then radiate from the location of change as small artificial waves through the system. Consequently, procedures had to be designed to decrease the generation of these disturbances and to suppress them rapidly during propagation.

In this model the problem was approached as follows. If at a particular grid point, any of the four transport cross sections decreases to less than a preset value, then that grid point is taken out of the computation. Since the preset value is always greater than zero, this is a more restrictive condition than the two described above. This cross section search is made at intervals larger than the time step so that the computational noise generated by the boundary change decays. By searching at intervals larger than the time step and by using a more restrictive condition, a large proportion of the boundary changes are made by this procedure and therefore the effect of local discontinuities generated by the discrete boundary changes decrease.

In the flooding procedure each of the four surrounding grid points adjacent to a dry grid point is checked to verify whether it is under water. If one or more of the surrounding grid points are wet, then the water levels of those surrounding grid points that are under water are averaged. If this average water level is larger than the water level retained on the dry grid point, then the grid point may be allowed to flood. First a check is made on the transport cross sections between the dry grid point and each of the surrounding grid points that are under water. If any of these cross sections are negative, the point remains dry. If all are positive, then the grid point is allowed to flood and is added back to the computational field. The water level is set at the value which remains over the grid point when it becomes dry. This procedure preserves the conservation of water mass.

The check for flooding is also made at intervals larger than the time step, to reduce the effect of the discrete boundary position change when a grid point is added back into the computation.

**MODEL CALIBRATION**

The calibration was performed adjusting the bottom friction coefficient for the entire lagoon, through the comparison between measured and predicted time series of sea surface elevation (SSE) for 17 stations distributed throughout the main channels of the lagoon (Figure 1). For these stations were available time series of SSE of at least 30 days length, measured from 2002 to 2004 (ARAÚJO, 2005).

From the observation of the SSE data and from previous modelling efforts it was concluded that friction dissipates energy as the tidal wave propagates landward from the lagoon mouth. This behaviour is a common feature in estuarine environments (HSU et al., 1999) and was analysed and quantified in Ria de Aveiro by DIAS and FERNANDES (2006). Considering the lagoon geometry and the tidal range at the mouth, the magnitude of the bottom friction coefficient determines the tidal range variation along the lagoon channels. The remaining parameter subjected to adjustment during model calibration is therefore the bottom friction in the Manning-Chézy formulation for the bottom stresses (Eq. (5)). Prior modelling experiments (BURAU and CHENG, 1988) as well as results of data analysis (CHENG and GANTNER, 1985) suggests a stronger influence of the water depth in the bottom stresses than the Manning-Chézy relation does. This effect can be introduced into the computations by allowing Manning’s coefficient $n$ to vary as a function of water depth (CHENG et al., 1993). In this model, as a first approach the Manning’s $n$ values are assigned to a range of water depth rather than assigning them to every point. The relation adopted between $h$ and $n$ (Table 1) was based in the values presented by DIAS and LOPES (2006b) and
in the assumption that the flow must be damped in the intertidal areas of Ria de Aveiro.

In this work the calibration of the hydrodynamic model was performed in two steps. A qualitative calibration was initially made through a direct comparison between simulated and observed time series of SSE for the 17 stations represented in Figure 1. Through an iterative process the model was re-run in order to optimise the adjustment between the results. In this process the values of the Manning coefficient previously referred were altered by a multiplicative factor for several blocks defined in the numerical bathymetry. This is a repetitive procedure that finishes when it is obtained the best possible adjustment between the simulations and the observations. Figure 2 shows the best results for 6 of these stations.

After obtaining the best adjustment, the next step is the quantification of the model accuracy. With this purpose it was performed a quantitative comparison, through the determination of the deviations between the model results and the observations. With this purpose were computed the relative mean absolute error (RMAE), the root-mean square (RMS) and the Skill.

The RMAE is being used to evaluate numerical model results by several authors (for instance Fernandes et al. (2001)) and was defined by Walstra et al. (2001):

\[
RMAE = \frac{\sum |\zeta_{\text{mod}}(t) - \zeta_{\text{obs}}(t)|}{N_{\text{mod}}}
\]

(6)

where \( \zeta_{\text{mod}} \) and \( \zeta_{\text{obs}} \) are the averages of the predicted and observed SSE, respectively. The qualification for RMAE ranges suggested by Walstra et al. (2001) is presented in Table 2.

The RMS of the difference between the observed and predicted surface elevation is also used by numerous authors (for instance Dias and Lopes (2006a,b)) to evaluate models accuracy:

\[
\text{RMS} = \left[ \frac{1}{N} \sum_{i=1}^{N} (\zeta_{\text{obs}}(t) - \zeta_{\text{mod}}(t))^2 \right]^{1/2}
\]

(7)

where \( \zeta_{\text{obs}}(t) \) and \( \zeta_{\text{mod}}(t) \) are the observed and the simulated SSE, respectively, and \( N \) is the number of measurements in the time series.

Model predictive skill is being recently used to evaluate all prognostic quantities and is based on the quantitative agreement between model results and observations (Warner et al., 2005):

\[
\text{Skill} = 1 - \frac{\sum (X_{\text{mod}} - X_{\text{obs}})^2}{\sum (X_{\text{mod}} - \bar{X})^2 + \sum (X_{\text{obs}} - \bar{X})^2}
\]

(8)

where \( X \) is the variable being compared with a time mean \( \bar{X} \). Perfect agreement between model results and observations will yield a skill of one and complete disagreement yields a skill of zero.

The RMAE, RMS and Skill were computed for each station and

<table>
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<tr>
<th>Table 1: Bottom friction coefficients</th>
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<tr>
<td>Water depth (m)</td>
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<tr>
<td>-----------------</td>
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<tr>
<td>-2.5 ( \leq h ) &lt; -2.0</td>
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<tr>
<td>-2.0 ( \leq h ) &lt; -1.5</td>
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<tr>
<td>-1.5 ( \leq h ) &lt; -1.0</td>
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<td>-1.0 ( \leq h ) &lt; -0.5</td>
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<tr>
<td>-0.5 ( \leq h ) &lt; 0.0</td>
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<tr>
<td>0.0 ( \leq h ) &lt; 0.5</td>
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<tr>
<td>0.5 ( \leq h ) &lt; 1.0</td>
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<tr>
<td>1.0 ( \leq h ) &lt; 3.0</td>
</tr>
<tr>
<td>3.0 ( \leq h ) &lt; 10.0</td>
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<tr>
<td>( h \geq 10.0 )</td>
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<tr>
<th>Table 2: Qualification of error ranges for RMAE</th>
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<tr>
<td>Qualification</td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Excellent</td>
</tr>
<tr>
<td>Good</td>
</tr>
<tr>
<td>Reasonable</td>
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<tr>
<td>Poor</td>
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<tr>
<td>Bad</td>
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Figure 2. Time series of SSE for stations C, D, F, H, J and L, used in the hydrodynamic model calibration (• data; - model).
In general there is a good agreement between the predicted and the observed SSE for all the stations (Figure 2 and Table 3), revealing the model’s ability to reproduce nature. The calculated values of Skill and RMAE reveal an excellent agreement between the prediction of the model and the observations. The RMS values may also be considered acceptable for all the stations, ranging from 0% to a maximum of 14% of the local amplitude.

However, there are some features that must be addressed. It is necessary to mention that the agreement between the predicted and the observed values at station D (mouth of the lagoon) is almost perfect. In fact, the tide synthesised for station D was specified at the ocean open boundary, with some corrections resulting from several models experiments. As the open boundary is located 2700 m westward of station D, a delay of about 5 min and 20 seconds was estimated between the tide at both locations as well as an amplitude correction factor of 1.004. This procedure allowed the specification of an accurate boundary condition and the minimisation of errors between the model results and observations for all the stations.

The best model results are obtained for the stations closer to the lagoon mouth, as station J (RMAE=0.00, RMS=0.02, Skill=1). The adjustment between the simulations and the observations may be considered excellent for all the stations located at the lagoon central area, with Skill values very close of the unity, RMAE negligible and RMS lower than 10 cm. These values may be considered as included in the natural uncertainties of the instruments used to measure the SSE.

The worse results are for the stations located at the far end of the channels, as A, E, G, K, L, P and Q. Although considered excellent by the RMAE classification, the skill is a little bit far from the unity and the RMS values are higher than 20 cm. This poorer reproduction of the reality by the model may be considered natural according to several factors. First, the natural discrepancies between model results and nature accumulate landward, as the tide propagates from the inlet. It also must be pointed out that these areas are very narrow and therefore these differences may be explained by an inaccurate definition of the numerical bathymetry, even defining cells of 40×40 m. For that it was impossible to improve the agreement between simulated and observed SSE. In the case of stations K and L, localised at Espinheiro channel and therefore under the direct influence of the run-off of Vouga river, may be the accuracy could be improved if the freshwater flow was known and considered in the simulations to force the model.

The comparison between the RMAE, RMS and Skill results suggest that RMAE is not an appropriate parameter to quantify the models accuracy in predictions of SSE. In fact, according to Tables 2 and 3 the results should be classified as excellent for all the stations and it is almost impossible to distinguish the models performance between the stations. According to the results, the parameter most sensible and intuitive and therefore considered as more adequate to quantify the models accuracy is the RMS.

The harmonic analysis is another quantification method used to perform the evaluation of models results in several studies (for instance DIAS and LOPES, 2006a,b). In this work this methodology was also applied to evaluate the model accuracy, determining the harmonic constants for the 17 stations previously referred. The software t_tide (PAWLÓWICZ et al., 2002) was used to analyse 30 day length time series of observed and predicted SSE for all the stations. Figure 3 shows the comparison between tidal amplitude and phase for the main harmonic constituents determined from the predicted and observed time series of SSE. The agreement between the predicted and observed values is rather good both in amplitude and in phase for the semidiurnal constituents, which are the major tidal constituents in Ria de Aveiro (DIAS et al., 2000). For the $M_2$ constituent, which amplitude is the largest, the mean difference between predicted and observed amplitudes is about 5 cm. The mean phase difference is 5°, which means that the average delay between the predicted and observed tide is about 10 minutes for these constituent. For the diurnal constituents the amplitude agreement may be considered good for all the stations as well as the phase agreement, except for stations E, F, M and N. The comparison between these values reveals that the amplitude of the major constituents may be considered well represented by the numerical model for the entire lagoon, with average differences lower than 2 cm. However, there are some features that must be referred. It is crucial to refer that the agreement between the predicted and observed values at station D (mouth of the lagoon) is not perfect, as it may be expected.

The results from the harmonic analysis show that constituents $M_2$ and $S_2$ together determine about 90% of the astronomic tide in Ria de Aveiro, in accordance with previous studies (DIAS et al., 1999; 2000).

CONCLUSION

The results obtained reveal a good agreement between predictions and measurements and therefore the hydrodynamic model for Ria de Aveiro may be considered successfully calibrated. It reproduces accurately the propagation of the tidal wave in Ria de Aveiro and therefore the barotropic flows in this complex lagoon. The model can therefore be used in the future to study important issues concerning the lagoon hydrodynamic and water quality.

When comparing these results with previous modelling efforts in Ria de Aveiro (DIAS and LOPES, 2006a,b), the refinement of the grid provide best results for the narrower channels of the lagoon, especially for Ilhavo channel.

The comparison between different methods to quantify the models accuracy revealed that RMS is the most sensible and intuitive parameter, as well as the deviation between the harmonic constants determined for the simulated and observed SSE.

LITERATURE CITED

Figure 3. Distributions of tidal amplitude and phase for $M_2$, $S_2$, and $K_1$, harmonic constituents (● data; - model).


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